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# Performance study of global solar radiation estimate's models for Uttar Pradesh

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**Abstract** : This work studies the performance of global solar radiation estimates correlation by using some statistical parameters, which enables the model tester to determine whether or not a model's predictions are statistically significant at a particular confidence level. Global solar radiation and the bright sunshine hour data have been analyzed to find the regression constants of the modified Angstrom linear correlation. The proposed correlation with regression constants $a = 0.300$ and $b = 0.407$ gives the better results.

**Keywords** : Global radiation, sunshine hour, $t$-statistics

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To achieve an optimally designed solar energy conversion system, we require knowledge about the solar radiation at a particular location. The best radiation information is obtained from experimental measurements of the global (total) and diffuse component of the solar insolation at the place. In many countries, bright sunshine hours are recorded at a number of places rather than total radiation. Different workers have developed many correlation based on meteorological parameters along with the sunshine hour data such as relative humidity, temperature, precipitation, latitude dependent geographical parameters *etc*. Several workers from many parts of the world [1–15], have examined the relationship between sunshine duration and global radiation. Angstrom [16] was the first to propose a linear correlation relating sunshine hour-global radiation which was later modified by Prescott [17] and Page [18] giving in a more convenient form of the correlation equation. This linear correlation between global solar radiation and the sunshine hour duration is the most popular one for its simpler form. We have analyzed the published meteorological data of global solar radiation and sunshine duration [19] for the city Lucknow, Uttar Pradesh, India. The location of the city is 26.75°N and 80.88°E in India.

The objective of this work is to find out the regression constants $a$ and $b$ of the modified Angstrom linear correlation by using standard least square analysis technique. Predicted results are compared with the correlations of Reitveld [2], Bahel *et al* [4], Alnaser [6] and Srivastava *et al* [19] along with the experimental observations by finding the statistical errors. The $t$-statistics is used simultaneously for the comparison and performance of solar radiation estimate's models.

The ratio $\overline{H}/H_0$ *i.e.* the experimental values of monthly mean daily global solar radiation $H$ and monthly average daily extra-terrestrial radiation $H_0$ on a horizontal surface and the possible sunshine hour $\overline{S}/S_0$ are taken from the literature [19] for Uttar Pradesh (UP), India. The values of global solar radiation were measured from April 1989 to March 1990 at Lucknow, which is centrally located in UP, and the climatic conditions of many big neighbouring cities do not differ appreciably. The global radiation was measured using NI precision pyranometer (No. 0266) manufactured by National Instruments Ltd., Calcutta, India. The calibration factor of NI precision pyranometer, used for the measurement of global solar radiation from sun and sky on a horizontal surface, was 8.41 $\mu$v/w/m$^2$. The details of the measurements can be found in Ref. [19].

**Table 1.** Regression parameters $a$ and $b$ used in different models.

| Correlations | Regression parameters $a$ | Regression parameters $b$ | Sky transmutivity $(a+b)$ |
|---|---|---|---|
| This work | 0 3000 | 0 4070 | 0.7070 |
| S K Srivastava *et al* [17] | 0.2006 | 0.5313 | 0.7319 |
| V Bahel *et al* [4] | 0.1750 | 0.5520 | 0 7270 |
| W E Alnaser [6] | 0.2843 | 0.4509 | 0.7352 |
| M R Reitveld [2] | 0.1800 | 0.6200 | 0.8000 |

For the prediction of global solar radiation, we have considered the most commonly used model is the linear regression of Angstrom as modified by Prescott [17], Page [18] and others. The equation correlates the monthly average daily global solar radiation $\overline{H}$ to the monthly average daily number of bright sunshine hours $\overline{S}$ by the linear function [3,13,14,20]

$$\frac{\overline{H}}{H_0} = a + b\left(\frac{\overline{S}}{S_0}\right), \tag{1}$$

$$H_0 = \frac{24}{\pi} I_{SC} E_0 \sin\phi \sin\delta\left[\frac{\pi}{180}\omega_S - \tan\omega_S\right], \tag{2}$$

$$\delta = \sum_{n=0}^{3} c_n \cos(nt) + \sum_{n=1}^{3} d_n \sin(nt), \tag{3}$$

$$t = \frac{2\pi}{365.24}(N_D - 80), \tag{4}$$

$$E_0 = 1 + 0.0335\cos\left(\frac{2\pi N_D}{365.24}\right), \tag{5}$$

$$\omega_S = \cos^{-1}(-\tan\phi\tan\delta), \tag{6}$$

$$S_0 = \frac{2}{15}\omega_S, \tag{7}$$

where $H_0$ is the monthly average daily extraterrestrial radiation on a horizontal surface, $I_{SC}$(= 1.367 kW/m$^2$) is the solar constant, $\phi$ is the latitude of the location, $\delta$ is the solar declination angle, $E_0$ is the eccentricity correction factor, $\omega_s$ is the hour angle, $S_0$ is the daylength and $N_D$ is the number of day sequence of the year starting from the first January. The values of coefficients $c_n$ and $d_n$ of eq. (3) are $c_0 = 0.386470$, $c_1 = -0.392624$, $c_2 = 0.377853$, $c_3 = 0.030124$, $d_1 = 23.259526$, $d_2 = 0.131544$, $d_3 = -0.167013$, taken from the literature [20]. The regression constants $a$ and $b$ of eq. (1) have been determined using standard regression technique. The regression constants obtained from different models are listed in Table 1.

The performance of the models used in this study will be discussed using some statistical errors namely the root mean square error (RMSE), mean bias error (MBE), mean relative percentage error (MRPE) and the $t$-statistics ($t$-STAT) given by the following expressions [3,14,21–24]

$$\text{RMSE} = \left[\frac{1}{n}\sum_{i}^{n}(P_i - M_i)^2\right]^{\frac{1}{2}}, \tag{8}$$

$$\text{MBE} = \frac{1}{n}\sum_{i}^{n}(P_i - M_i), \tag{9}$$

$$\text{MRPE} = \frac{1}{n}\sum_{i}^{n}\frac{P_i - M_i}{M_i}x100, \tag{10}$$

$$t-\text{STAT} = \left[\frac{(n-1)\text{MBE}^2}{\text{RMSE}^2 - \text{MBE}^2}\right]^{\frac{1}{2}}, \tag{11}$$

where $P_i$ and $M_i$ are the $i$-th predicted and measured values of $n$-observations. Generally, the lower the value of RMSE, the more accurate the model is. However, a few large errors in the sum, can produce a significant increase in RMSE. MBE test provides information with respect to over-estimation (positive value) or under-estimation (negative value), but the lower the absolute value, the better the model performance. To determine whether a model's estimation is statistically significant, one simply has to determine a critical value obtainable from standard statistical table *i.e.* $t_{\alpha/2}$ at the $\alpha$ level of significance and ($n$ – 1) degrees of freedom. For medel's estimates to be judged statistically significant at the (1 – $\alpha$) confidence level, the predicted $t$-value must be less than the critical value $t_C$. The smaller the value of $t$, the better is the model performance.

Regression constants $a$ and $b$ of eq. (1) have been established by standard least square analysis technique using experimental observations of mean monthly daily radiation on horizontal surface and the sunshine records at Lucknow (UP). The value of correlation coefficient is $r$ = 0.906. The regression constants obtained from different correlation are listed in Table 1. Predicted results are compared with the correlations obtained by Reitveld [2], Bahel *et al* [4], Alnaser [6] and Srivastava *et al* [19] and with the experimental observations. For this purpose, the statistical errors (RMSE, MBE and MRPE) are evaluated and the results are presented in Table 2. The entries in Table 2 indicate that this work predicts global solar radiation more accurately, since the values of statistical errors are minimum.

**Table 2.** Values of statistical errors estimated from different correlations.

| Correlation | RMSE | MBE | MRPE% | $t$-STAT | Critical $t_c$ |
|---|---|---|---|---|---|
| This work | 0.279 | –0.001 | 0.190 | 0.012 | 3.106 |
| Srivastava *et al* [19] | 0.342 | –0.073 | 1.686 | 0.722 | 3.106 |
| Bahel *et al* [4] | 0.386 | –0.165 | –3 537 | 1.565 | 3.106 |
| Alnaser [6] | 0 325 | 0.146 | 2.826 | 1.664 | 3.106 |
| Reitveld [2] | 0.543 | 0.322 | 5.531 | 2.504 | 3 106 |

There are some drawbacks in RMSE and MBE errors. In RMSE test, a few large errors in the sum can produce a significant increase in RMSE and does not differentiate between under-estimation and over-estimation. In MBE test, over-estimation of an individual observation will cancel under-estimation in a separate observation. So, these test may not be an adequate indicator of a model's performance. It is possible to have a large RMSE value and at the same time a small MBE or a relatively small RMSE and a large MBE. Another drawback of using the RMSE and MBE that the dimension values of the indicators do not allow model testing under various meteo-climatic conditions [24]. To circumvent these problems, a relationship for $t$-statistics has been developed as a function of widely used RMSE and MBE.

Table 2 summarizes the statistical performance of the correlations examined in this analysis. It is seen that this work has the minimum value of $t$-STST and less than the critical value $t_c$ in all cases and the present work has the better performance than the others. The correlations of the present work and of Srivastava *et al* [19], have the regional character and the other correlations (Bahel [4], Alnaser [6] and Reitveld [2]) are believed to have the universal characters. Among these three correlations, Bahel *et al* [4] yields lower MBE, RMSE and $t$-STAT values while the other models follow. However, the standard correlation exhibits a tendency for over-estimation. Table 2 also indicates that the results are statistically significant at the particular confidence level ($1 - \alpha = 1 - 0.01 = 99\%$), since the $t$-STAT values are less than the critical $t_c$. The critical $t_c$ value can be obtained from standard statistical table [25].

The monthly values of deviation in percentage are also evaluated by MRPE and listed in Table 3. It is seen that in the month of March, the deviation is maximum for all the cases but this work has the lower maximum deviation value and for rest of the months, the value of deviation lies within the limit of 7.0%. Yearly mean deviation (Table 2) lie in the limit less than 5%.

**Table 3.** Monthly values of relative percentage errors (%) obtained from different models.

| Month | Relative percentage errors (%) | | | | |
|---|---|---|---|---|---|
| | This work | Srivastava [19] | Bahel [4] | Alnaser [6] | Reitveld [2] |
| 01 | 1 714 | 0.605 | 2.013 | 10.332 | 4.945 |
| 02 | 0.815 | 0.854 | 2.048 | 10.716 | 4.258 |
| 03 | 12 445 | 14 752 | 15 686 | 26.193 | 16.749 |
| 04 | –1.494 | –2 859 | –1.445 | 6.504 | 1.574 |
| 05 | –1.350 | 2 930 | –1 476 | 6.406 | 1.679 |
| 06 | –0.968 | –10.373 | –7.536 | –2 523 | 0 452 |
| 07 | 3.782 | –8.288 | –4.925 | –0.493 | 0.811 |
| 08 | 2.242 | –10.098 | –6.705 | –2.505 | 3 163 |
| 09 | 1.266 | –5.383 | –3.005 | 3.222 | 3.334 |
| 10 | –5 772 | –6.806 | –5.501 | 2.204 | –2.781 |
| 11 | –7 227 | –7.660 | –6.479 | 1.324 | –4.161 |
| 12 | –3.177 | –4.257 | –2.913 | 4.997 | –0 106 |

The present study revises the work of Srivastava *et al* [19] for Lucknow (UP) and finds a better correlation of modified Angstrom type linear correlation with regression constants $a = 0.300$, $b = 0.407$ and the correlation coefficient $r = 0.906$. Comparison and performance test of the models is done simultaneously by using a new statistical indicator the $t$-statistics ($t$-STAT). Analysis shows that among the examined correlations, this work predicts the global solar radiation more accurately with a deviation of 1–7%.

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